SIMILARITY CRITERIA FOR SHOCK WAVES OF PLASMODYNAMIC DISCHARGES FROM MAGNETOPLASMA COMPRESSORS IN DENSE GASES

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Pulse plasma accelerators, functioning as electromagnetic focusers of plasma flows, and magnetoplasma compressors (MPCs) are used for solving a series of scientific and applied problems [1-4]. Of practical interest is the erosion MPC [4-6] where, unlike the gas discharge MPCs described in [7-9], the plasma is formed from vaporization products of the construction elements (usually low-melting-point dielectric rings) and is accelerated under quasisteady-state conditions, resulting in dense, high-velocity (about 30-70 km/sec) plasma flow. If an erosion MPC operates in a gas medium, the erosion plasma flow decelerates in the dense gas filler. A zone of shock-compressed plasma then forms with an internal energy on the order of the enthalpy of the flow's deceleration. A "plasma piston" pushes back the gas-filler, creating a shock wave in the gas. In [10, 11], it was shown how erosion MPC discharges in gases can be used for studying radiative gasdynamics and kinetic processes by serving as generators of shock waves, as sources of visual and ultraviolet radiation, etc. An experimental study of such discharges was conducted in [6, 10, 11].

This analysis, based on numerical research into the phenomenon considered above, allows us to determine similarity criteria and obtain relations relating instrument characteristics to the parameters of both the plasma formations and the shock waves which are formed due to such discharges. The results of this analysis can be used for studying and generalizing experimental data and for creating new, special-purpose, electrophysical devices based on MPC discharges.

1. Calculation Model. The numerical model in [12] was used for calculating the time dependences of the plasma flow parameters: the acceleration of the plasma was described using an electrodynamic approximation which takes into account the triviality of the acceleration zone's length; the energy transfer from the storage device was determined using Kirchhoff's laws; and the erosive output of mass was determined by taking into account the persistance of the plasma formation. The corresponding system of equations has the form

$$\begin{gathered} v_0 = L'I^2/\dot{m}, \quad R_e = \dot{L}'v_0/2\eta_*, \\ L_p \frac{d}{dt} I + (R_e + R_p) I = U, \quad \frac{d}{dt} U = -I/C, \\ \dot{m} = m_* \langle A \rangle (0.25|I| + 0.64|I^2| + 0.81|I^3|) - T_* \frac{d}{dt} \dot{m}, \end{gathered} \tag{1.1}$$

where v_0 is the velocity; m is the plasma flow rate in mg/μsec; I is the discharge current in MA; L' is the linear inductance of the electrodes; R_e is the resistance of the MPC; R_p and L_p are the active and inductive resistances of the circuit; U is the voltage on a storage device of capacitance C; <A> is the atomic weight of the plasma ions; m_* is the intensity coefficient of the plasma formation; T_* is the inertia of plasma formation; and η_* is the kinetic efficiency.

For calculations, we considered both the time for the plasma to go from the MPC to the shock deceleration zone and the related changes in the velocity and density profiles of the jet.

The effective radial dimension of the plasma flow's leading section R, which is necessary for calculating the flow density ρ_0, is determined using the experimental data in [10, 11]. It is significant that for a wide range of parameter, the quantity R remains almost constant due to focusing of the flow and is basically determined by the radial dimensions of the external and internal electrodes of the accelerator R_1 and R_2; for making estimations, one can assume that $R \cong 0.35\ (R_1 + R_2)$.

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Deceleration of the plasma flow in the gas is described using the approach considered in [10]. According to [10], the shock wave consists of the following fundamental zones: the flow of erosion plasma formed by the MPC, the shock-compressed erosion plasma, the shock-compressed gas, and the cold, unperturbed gas. The expressions for the shock wave velocities in the erosion plasma and the gas, D_1 and D_2, for the velocity of the contact boundary (cb) between the erosion products and the gas-filler v_{cb}, and for the pressure p behind the shock wave fronts have the forms

$$v_{cb}=v_0/\beta,\quad \beta=1+\sqrt{\rho_g/\rho_0},\quad D_1=v_{cb}[\gamma_p+1-\beta(\gamma_g-1)]/2,$$
$$D_2=v_{cb}(\gamma_g+1)/2,\quad p=\frac{\gamma_g+1}{2}\rho_0(v_0-v_{cb})^2, \tag{1.2}$$

where ρ_g is the density of the cold gas, and γ_g is the adiabatic efficiency index of the gas (for normal air pressure and for characteristic "plasma piston" velocities of v_{cb} = 5-15 km/sec, $\gamma_g = 1.2 \pm 0.02$) [13]. The adiabatic efficiency index of the erosion plasma γ_p is usually γ_p = 1.05-1.3; it is evident from relations (1.2) that such an uncertainty in the values of γ_p does not have a significant effect on the fundamental shock wave parameters.

We will also consider the dynamic pressure which is exerted on the wall due to the deceleration of a moving, dense layer of shock-compressed gas by a solid barrier, $p^* = p(3\gamma_g - 1)/(\gamma_g - 1)$, a quantity which is of interest for a series of practical applications.

2. Comparison with Experiment. Correlation of the model described above with the actual processes which occur in such discharges is performed by comparing the results of calculations and experiments [10, 11] for a series of MPC operating modes: MPC discharges in air of normal density, dimensions of the electrode system — $2R_1$ = 10 mm, $2R_2$ = 40 mm — a storage device capacitance of 900 μF, and an initial voltage of 3-5 kV. Fluoroplastic served as a plasma-forming substances, where $\eta_* = 0.8$, $T_* = 1$ μsec, $m_* = 0.15$.

Some of the parameters for an MPC discharge and for the plasma flow are given in Fig. 1 with U_0 = 5 kV. Because of the decrease in flow velocity with time (curve 1), the maximum value for the plasma density before the shock wave (curve 2) is displaced relative to the maximum value of the discharge current (curve 3), and the maximum value of the flow velocity pressure (curve 4) is attained somewhat after the maximum value of the kinetic discharge power (curve 5).

The space–time parameters of the shock wave formed during discharge are given in Fig. 2: Curves a are the calculated velocities; curves b show the positions of the shock wave in the gas 1 of the contact boundary between the erosion products and the gas 2, and of the shock wave in the erosion plasma 3.

At the beginning of the discharge, the velocity pressure is insufficient for overcoming the opposition of the medium, and a layer of erosion plasma squeezes to the surface of the dielectric. An increase in the velocity pressure of the flow allows the shock wave to exit from the MPC with a passage time of $t_{1/2}$. As a result, the time for the shock interaction t'_i = 35 μsec substantially exceeds the time for energy release.

Comparison of calculation and experimental data [10, 11] for the discharge current (point 6, Fig. 1), for the plasma velocity flow (point 7, Fig. 1), and for the position of

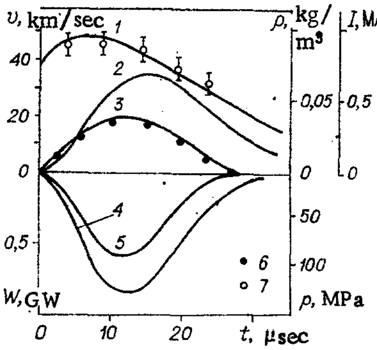

Fig. 1

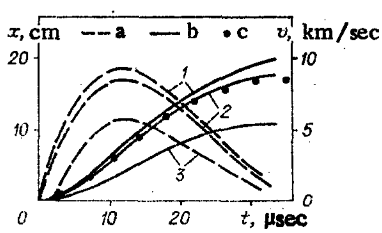

Fig. 2

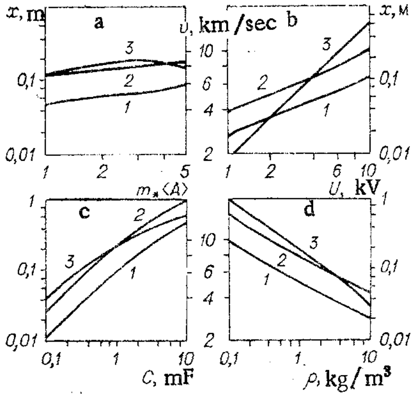

Fig. 3

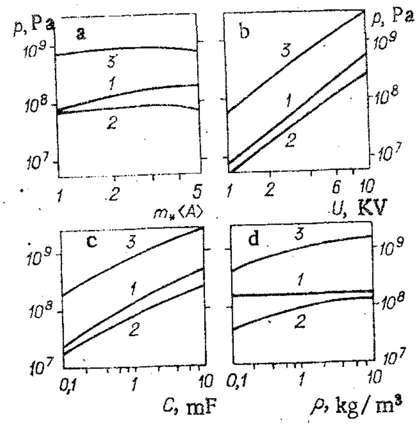

Fig. 4

the contact boundary (curve c, Fig. 2) shows that this model reliably describes the dynamics of the energy contribution to the discharge of the erosion plasma flow parameters and gives the correct values for the maximum velocity of the "plasma piston" $v'_m = 8.5$ km/sec and for the characteristic longitudinal dimension of the forced back gas volume for a current half-period of $x'_{\frac{1}{2}} = 18$ cm. The time at which the shock wave intensity is at a maximum, and the corresponding position of the contact boundary ($t'_m = 12$ μsec, $x'_m = 7$ cm) also agree well with the experiments in [10, 11]. A comparison of calculated and experimental data for the different operating modes indicated above leads to the same conclusions.

It follows that this model sufficiently well describes the fundamental plasma dynamical features of erosion MPC discharges in gases. One can, therefore, use this model for multiparametric optimization of engineering calculations and for analyzing the effects of instrument characteristics on the fundamental shock wave parameters.

3. Effect of Instrument Characteristics on the Shock Wave Parameters. Calculations using this model were made for a wide range of changes in the energy contribution to the discharge, of electrical circuit characteristics, of gas-filler properties, and of the dimensions of the MPC working substance. The dependences for a number of parameters which characterize this phenomenon are given in Figs. 3 and 4. The distance by which the gas filler is forced back by the erosion products for a half-period $x_{\frac{1}{2}}$, the maximum velocity of the contact boundary v_m and its position at this moment in time x_m (corresponding to curves 1-3 in Fig. 3), the maximum velocity pressure of the plasma flow $\rho_{0m}v_{0m}^2$, the maximum value of the pressure behind the shock wave front p_m, and the maximum value of the dynamical pressure p^*_m corresponding to curves 1-3 in Fig. 4.

An analysis of these dependences was done using more detailed available data, showing the following.

The shock wave parameters weakly depend on the intensity coefficient of the output of eroded mass $m_*\langle A\rangle$ (Figs. 3a and 4a), although the latter has a substantial effect on the quantity of accelerated mass flow m and on the plasma velocity v_0 [12]. This is explained as follows: The shock wave parameters basically depend on the working velocity pressure $\rho_0 \cdot (v_0 - v_{cb})^2(\gamma_p + 1)/2$. For constant MPC kinetic power $\dot{m}v_0^2/2$, the velocity pressure $\dot{m}v_0$ increases in proportion to $\dot{m}^{0.5}$ with an increase in $\dot{m}$ (and a decrease in v_0). On the other hand, for a decrease in v_0 to a level $v_0 \sim (2\text{-}5)\cdot v_{cb}$, the working velocity pressure becomes noticeably smaller than $\dot{m}v_0$, and the kinetic power and efficiency of energy transfer from the storage device [12] decrease. These effects compensate one another to a significant degree under the conditions considered above, and the shock wave parameters depend weakly on the MPC working substance.

Increasing the energy contribution to the discharge causes an increase in the scale and the dynamical characteristics of the shock wave. However, for increasing the energy contribution E_0 by increasing the capacitance of the storage device, the efficiency in forcing back the gas is much higher than would be the case when E_0 is increased due to an increase in the initial voltage, i.e., an increase in power for $t_{\frac{1}{2}} = \text{const}$ (see Figs. 3b and 4b). This is explained by an increase in the shock wave interaction time and a decrease in gas backpressure.

Increasing the density of the gas ρ_g causes the "plasma piston" to advance more slowly (see Figs. 3d and 4d). For low densities, the efficiency of the working velocity pressure decreases due to high v_{cb} (which decreases the pressure and the rate of increase of v_{cb}), and the shock wave interaction time increases due to large $x_{\frac{1}{2}}$ and due to transit effects (which results in an even larger increase in $x_{\frac{1}{2}}$).

4. Similarity Criteria. Analysis of the calculated data shows that a change in one of the factors can be compensated by a change in the others and, hence, the phenomena are qualitatively and quantitatively similar. This allows one to determine similarity criteria, which determine the character of the processes, and to obtain generalized criteria dependences which quantitatively describe the effects of erosion MPC discharges in gases.

The density of the gas-filler and the radial dimension of the flow's leading section enter into the equations of the model described above only in the combination $m_p = \rho_g \pi R^2$, which is the linear mass of the "raked-up" gas.

An analysis of the calculations made with changing energy contributions by varying the capacitance and the voltage on the storage device shows that the maximum velocity of the contact boundary v_m is a function of the maximum power of the energy contribution W_m. This is because the value of v_m is determined by the working velocity pressure of the plasma jet in the shock wave, where the latter depends practically linearly on the electrical power of the discharge. The pressures p_m and p^*_m are also functions of the maximum power.

The distance $x_{\frac{1}{2}}$ is almost proportional to $v_m t_i$, and x_m is proportional to $v_m t_m$. Here, the shock-wave interaction time is $t_i \cong t_{\frac{1}{2}} + x_{\frac{1}{2}}/\langle v_0 \rangle$; the time at which the shock-wave parameters are at their maxima is $t_m \cong t_W + x_m/\langle v_0 \rangle$; t_W is the time at which the electrical power of the MPC discharge is at a maximum for capacitance storage devices the energies at times $t_{\frac{1}{2}}$ and t_m can be found using the equations for an oscillatory circuit with $R_e = L' \langle v_0 \rangle / 2\eta_*$; $\langle v_0 \rangle$ is the average velocity of the plasma flow as determined from (1.1). Such similarity is related to the similar kinetic power profiles of the MPC for a discharge half-period.

Calculations showed that, in the first approximation, one can neglect an explicit dependence of these parameters on $m_* \langle A \rangle$. Therefore, for erosion MPC discharges in gases, the values of the shock-wave parameters are basically determined by the linear mass of the gas m_p, by the maximum electrical power W_m, and by the times t_i and t_m (and also by the adiabatic index of the gas γ_g).

This allows one to generalize the results of the given calculations in the forms of criteria relations for characteristic parameters.

The obtained dependences have the form

$$x_{1/2}/x'_{1/2} = (W_m/W'_m)^{0.42} (m_p/m'_p)^{-0.4} (t_i/t'_i)^1; \quad (4.1)$$

$$v_m/v'_m = (W_m/W'_m)^{0.42} (m_p/m'_p)^{-0.4}; \quad (4.2)$$

$$x_m/x'_m = (W_m/W'_m)^{0.42} (m_p/m'_p)^{-0.4} (t_m/t'_m)^1; \quad (4.3)$$

$$p_m/p'_m = (W_m/W'_m)^{0.8} (m_p/m'_p)^{0.25}; \quad (4.4)$$

$$p^*_m/p^{*\prime}_m = (W_m/W'_m)^{0.8} (m_p/m'_p)^{0.25} (\Gamma/\Gamma')^1, \quad \Gamma \equiv (3\gamma_g - 1)/(\gamma_g - 1) \quad (4.5)$$

(the primes denote the parameters considered earlier for discharge conditions). Calculations using (4.1)-(4.5) are most reliable for minor deviations from the "base" mode.

One should note that using (4.1)-(4.5) allows one to make estimations both when the MPC operates with a capacitance storage device and when it operates with other types of generators.

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NUMERICAL ANALYSIS OF GAS FLOW TAKING INTO ACCOUNT RESISTANCE FORCES

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1. To describe the questions of motion for a gas moving along pipes and arterial fissures and for gas being filtrated through a porous medium, one usually uses empirical laws (D'Arcy's law, the Forkhgeymer relation, etc. [1, 2]).

The system of equations for gasdynamics taking into account resistance forces is given in [3, 4], and an analysis of a system of general, quasilinear equations was conducted in [5].

This study considers the flow of gas which is described by the Euler equation with resistance forces.

The system of equations which describes the flow of isothermal gas has the form

$$\frac{\partial}{\partial t}\rho + r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}\rho u) = 0; \qquad (1.1)$$

$$\frac{\partial}{\partial t}j + r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}ju) = -\frac{\partial p}{\partial r} - F; \qquad (1.2)$$

$$p = c^2\rho, \qquad (1.3)$$

where p is the pressure; ρ is the density; u is the velocity of the flow; $j = u\rho$ is the density of the gas flow; c is the isothermal velocity of sound; F is the force of resistance to the gas flow; ν is a symmetry index (ν = 0 for the two-dimensional problem, ν = 1 for the cylindrical problem, and ν = 2 for the spherically symmetric problem); r is the position; and t is the time.

For small Reynold's numbers ($\mathrm{Re} = \lambda\rho u m\mu^{-1} \leq 1$)

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